2021

MATHEMATICS — HONOURS

Paper : CC-7

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

$\mathbb{R}$ denotes the set of real number.

Group – A

(Marks : 20)

1. Answer the following multiple choice questions with only one correct option. Choose the correct option and justify : (1+1)×10

(a) The singular solution of the equation, $y = \frac{2}{3}x\frac{dy}{dx} - \frac{2}{3x}\left(\frac{dy}{dx}\right)^2$, $x > 0$ is

(i) $y = \pm x^2$ (ii) $y = \frac{x^3}{6}$ (iii) $y = x$ (iv) $y = \frac{x^2}{6}$.

(b) Let $y_1(x)$ and $y_2(x)$ be two linearly independent solutions of the differential equation $x^2y''(x) - 2xy\,y'(x) - 4y(x) = 0$ for $x \in [1, 10]$. Consider the Wronskian
$w(x) = y_1(x)y_2{}'(x) - y_2(x)y_1{}'(x)$.

If $w(1) = 1$, then $w(3) - w(2)$ equals to

(i) 1 (ii) 2 (iii) 3 (iv) 5.

(c) If $x^2 + xy^2 = C$, where $C \in \mathbb{R}$, is the general solution of the exact differential equation $M(x, y)dx + 2xy\,dy = 0$, then $M(1, 1)$ is

(i) 3 (ii) 2 (iii) 4 (iv) 1.

(d) If $x^h y^k$ is an I.F. of the differential equation, $y(1 + xy)dx + x(1 - xy)dy = 0$, then the ordered pair (h, k) is equal to

(i) $(-2, -2)$ (ii) $(-2, -1)$ (iii) $(-1, -2)$ (iv) $(-1, -1)$.

(e) The integrating factor of the differential equation $\frac{dy}{dx}(x\log x) + y = 2\log x$ is

(i) $\log x$ (ii) e^x (iii) $\log(\log x)$ (iv) x.

(f) Singular solution of the equation $y = px + \frac{a}{p}$ where $p \equiv \frac{dy}{dx}$, is

(i) $\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$ (ii) $y^2 = -4ax$ (iii) $y^2 = 4ax$ (iv) $x^2 = 4ay$.

(g) Using variation of parameters the Wronskian of the following equation $y'' - 2y' + 1 = (x+1)e^{2x}$ is

(i) xe^{2x} (ii) e^{2x} (iii) e^{x} (iv) e^{-2x}.

(h) Particular integral of $\left(D^2 - 3D + 2\right)y = e^{5x}$ is

(i) $\frac{e^{5x}}{12}$ (ii) $\frac{e^{5x}}{13}$ (iii) $\frac{e^{5x}}{14}$ (iv) $\frac{e^{5x}}{15}$.

(i) The double limit $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

(i) exists and equal to 0 (ii) exists and equal to 1

(iii) exists and equal to 2 (iv) does not exist.

(j) Consider the vector field $\vec{F} = (ax + y + a)\hat{i} + \hat{j} - (x + y)\hat{k}$, where '$a$' is a constant. If $\vec{F}.\operatorname{curl}\vec{F} = 0$, then the value of a is

(i) -1 (ii) 0 (iii) 1 (iv) $\frac{3}{2}$.

Group – B

(Marks : 30)

Answer ***any six*** questions. 5×6

2. Find the family of curves such that, at any point of any member of the family, the x-intercept of the corresponding tangent line equals the ordinate at that point. 5

3. (a) Solve $\left(x^2y - 2xy^2\right)dx - \left(x^3 - 3x^2y\right)dy = 0.$

(b) Check whether the following equation is exact or not :

$$\left(2x^2 + 3x\right)\frac{d^2y}{dx^2} + (6x+3)\frac{dy}{dx} + 2y = (x+1)e^x.$$ 3+2

4. Find the general solution of the differential equation $y(4x+y)dx-2\left(x^2-y\right)dy=0.$ 5

5. Reduce the equation $xp^2 - 2yp + x + 2y = 0$ to Clairant's form by using the substitution $x^2 = u$ and $y - x = v$ and then solve it. 5

6. Solve by the method of variation of parameters, the equation $\dfrac{d^2y}{dx^2}-4y=\sin h\,x.$ 5

7. Solve the equation $\dfrac{d^2y}{dx^2}+(x-1)^2\dfrac{dy}{dx}-4(x-1)y=0$ in series about the point $x = 1$. 5

8. Solve for x and y :

$$\frac{dx}{dt}+\frac{2}{t}(x-y)=1$$
$$\frac{dy}{dt}+\frac{1}{t}(x+5y)=t.$$ 5

9. Solve by the method of undetermined coefficients

$$\frac{d^2y}{dx^2}+5\frac{dy}{dx}+4y=8x^2+3+2\cos 2x.$$ 5

10. Find a power series solution of the initial value problem

$$\left(x^2-1\right)\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+xy=0$$
$$y(0)=4$$
$$y'(0)=6.$$ 5

11. Solve by the method of variation of parameters, the equation $\dfrac{d^2y}{dx^2}+a^2y=\sec ax.$ 5

Group – C

(Marks : 15)

Answer ***any three*** questions. 3×5

12. Let $f(x, y)$ be continuous at an interior point (a, b) of domain of definition of f and $f(a, b) \neq 0$. Show that $f(x, y)$ maintains same sign in a neighbourhood of (a, b). What can you say about the sign of f in a neighbourhood of (a, b) if $f(a, b) = 0$? 3+2

13. Examine for existence of maxima or minima of the function $h(x, y)=x^2+y^2+(x+y+1)^2.$ 5

14. Find the maximum or minimum of the function $f(x, y) = xy$, subject to the condition $5x + y = 13$, using the method of Lagrange's Multipliers. 5

15. For the function $f(x, y) = (|xy|)^{1/2}$, show that both f_x and f_y exist at (0, 0) but is not differentiable at (0, 0). 2+3

16. Let $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x, y) = \frac{xy^2}{x+y}, \text{ if } x + y \neq 0$$

$$= 0 \quad \text{if } x + y = 0$$

Then find the value of $\left(\frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 f}{\partial y \partial x}\right)$ at the point (0, 0). 5

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